

Roll No.

DD-2806

M. A./M. Sc. (Final) EXAMINATION, 2020

MATHEMATICS

(Compulsory)

Paper First

(Integration Theory and Functional Analysis)

Time : Three Hours

Maximum Marks : 100

Note : Attempt any *two* parts from each question. All questions carry equal marks.

1. (a) State and prove Radon-Nikodym theorem.
(b) Let E be a measurable set such that $0 < \nu(E) < \infty$, then prove that there is a positive set A contained in E with $\nu(A) > 0$.
(c) State and prove Caratheodory extension theorem.
2. (a) State and prove Riesz-Markoff theorem.
(b) Define Baire set and prove that every compact Baire set is a G_δ set.
(c) If μ is finite Baire measures on the real line, then show that its cumulative distribution function F is monotone increasing bounded function which is continuous on the right, moreover $\lim_{n \rightarrow \infty} F(X) = 0$.

(A-28) P. T. O.

3. (a) Let $\{x_n\}$ be a weakly convergent sequence in a normed linear space X . Then prove that :
- The weak limit of $\{x_n\}$ is unique.
 - The sequence $\{\|x_n\|\}$ is bounded.
- (b) Let $C[a, b]$ be a function space of real valued continuous functions defined on $[a, b]$. Define $\|\cdot\|_\infty$ by $\|\cdot\|_\infty : C[a, b] \rightarrow \mathbb{R}$, where $\|f\|_\infty = \max_{t \in [a, b]} |f(t)|$, then prove that $(C[a, b], \|\cdot\|_\infty)$ is Banach space.
- (c) State and prove Riesz lemma.
4. (a) Prove that a Banach space is reflexive if and only if its dual space is reflexive.
- State and prove closed graph theorem.
 - State and prove Uniform Boundedness theorem.
5. (a) State and prove the generalized Lax-Milgram theorem.
- Prove that product of two bounded self-adjoint operators S and T on a Hilbert space H is self-adjoint iff the operators commute i. e. if $ST = TS$.
 - A Banach space is Hilbert space iff its norm satisfies the parallelogram law.