ED-762

M.A./M.Sc. 4th Semester
Examination, May-June 2021

MATHEMATICS

Paper - I

Functional Analysis-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove closed graph theorem.

(*b*) Let X be a Banach space and Y be a normed linear space. Let $\{T_i\}$ be a non-empty set of continuous linear transformation from X into Y, such that $\{T_i(x)\}$ is bounded for each x and X, then show that is $\{\|T_i\|\}$ is bounded.

(*c*) Let T be a bounded linear transformation from a Banach space X into a normed linear space Y. Then show that the openness of T implies the completness of Y.

Unit-II

2. (*a*) Let X and Y be normed linear space. Then show that $B(X, Y)$ the set of all bounded linear transformations from X into Y, is a normed linear space.

(*b*) Let X is a Banach space. Then show that X is reflexive if and only if X^* is reflexive, where X^* is the conjugate space of a normed linear space X.

(*c*) Let E be a real normed linear space and let M be a linear subspace of E. If $f \in M^*$, then show that there is a $g \in E^*$ such that $f \subset g$ and $\| g \| = \| f \|$.

Unit-III

3. (*a*) State and prove Bessel's inequality.

(*b*) If X is an inner product space and $x, y \in X$, then show that $|(x, y)| \leq \|x\| \, \|y\|$.

(*c*) Show that a Banach space is a Hilbert space if and only if the parallelogram law holds.

Unit-IV

4. (*a*) State and prove Riesz Representation theorem.

(*b*) Prove that every Hilbert space is reflexive.

(*c*) Let T be an operator on a Hilbert space H. Then there exists a unique operator T^* on H such that

$$(Tx, y) = (x, T^*y)$$

for all $x,\ y \in H$.

Unit-V

5. (*a*) If T_1 and T_2 are self-adjoint, then show that $T_1 T_2$ is self-adjoint if and only if they commute, i.e. $T_1 T_2 = T_2 T_1$.

(*b*) State and prove generalized Lax-Milgram theorem.

(*c*) If T is a normal operator on a Hilbert space H and D is any scalar, then show that $T\text{-}\lambda I$ is also normal.

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MATHEMATICS

Paper - III (C)

Fuzzy Set Theory and Its Applications-II

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Define fuzzy propositions with properties and examples.

(*b*) Define fuzzy quantifiers with examples.

(*c*) Write the method of inference from conditional and qualified propositions.

Unit-II

2. (*a*) Let f be a function defined by $f(a) = e^a$ for all $a \in [0, 1]$. Find the fuzzy intersection, fuzzy union, fuzzy implication and fuzzy compliment generated by f.

   (*b*) Explain approximate reasoning and fuzzy language with one such example.

   (*c*) Write the interpolation method and show that $B_2^1 \subseteq B_4^1 \subseteq B_1^1 = B_3^1$.

Unit-III

3. (*a*) Write a short note on design of fuzzy controllers.

   (*b*) Discuss possible ways of fuzzyfying the general dynamic system.

   (*c*) Discuss the design of a air conditioner fuzzy controller.

Unit-IV

4. (*a*) Define defuzzification and write any two methods of defuzzification.

(*b*) Aggregate graphically the fuzzy sets :

$$A_1 = \frac{0}{0}, \frac{.3}{1}, \frac{.3}{2}, \frac{.3}{3}, \frac{.3}{4}, \frac{0}{5}$$

$$A_2 = \frac{0}{3}, \frac{.5}{4}, \frac{.5}{5}, \frac{.5}{6}, \frac{0}{7}$$

$$A_3 = \frac{0}{5}, \frac{1}{6}, \frac{1}{7}, \frac{0}{8}$$

and solve it by the centroid method.

(*c*) Solve the following fuzzy linear programming problems

Max. $z = 6x_1 + 5x_2$

Subject to

$(5, 3, 2)x_1 + (6, 4, 2)x_2 \leq (25, 6, 9)$

$(5, 2, 3)x_1 + (2, 1.5, 1)x_2 \leq (13, 7, 4)$

$x_1 , x_2 > 0.$

Unit-V

5. (*a*) Let each individual of four decision makers has a total preference ordering $P_i\ (i \in N)$ on a set of alternatives $X = \{a, b, c, d\}$ as

$P_1 = (a, b, d, c)$; $P_2 = (a, c, b, d)$;

$P_3 = (b, a, c, d)$; $P_4 = (a, d, b, c)$

Find the fuzzy preference relation. Also find α-cuts of the fuzzy relation and group level of agreement concernng the social choice denoted by the total ordering (a, b, c, d).

(*b*) Explain individual and multiperson decision making in fuzzy environment.

(*c*) Explain construction of an ordering of all given alternatives by Shimura method.

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MATHEMATICS

Optional - A

Paper - IV

Operations Research

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) Use dynamic programming to solve

Minimize $z = p_1 \log p_1 + p_2 \log p_2 \; ..... + p_n \log p_n$

Subject to the constraints :

$p_1 + p_2 + p_3 + ... + p_n = 1$ and $p_j \geq 0 \; (j=1, 2, ... n)$

(*b*) What is principle of optimality ? Write the recursive equation approach to solve dynamic programming problem.

(*c*) Use dynamic programming to solve the following L.P.P.

Maximize $z = 3x_1 + 5x_2$

Subject to the constraints :

$x_1 \le 4,$

$x_2 \le 6,$

$3x_1 + 2x_2 \le 18$ and

$x_1,\ x_2 \ge 0$

Unit-II

2. (*a*) Consider a 'modified' form of 'matching biased wins' game problem. The matching player is paid ₹ 8 if the two coins turns both heads and ₹ 1 if the coins turns both tails. The non-matching player is paid ₹ 3 when two coins do not match. Given the choice of being the matching or non-matching player, which one would you choose and what would be your strategy ?

(*b*) Solve the following problem graphically :

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A & \begin{bmatrix} 3 & -3 & 4 \\ -1 & 1 & -3 \end{bmatrix} \end{array}$$

(*c*) For the following playoff matrix, find the value of the game and the strategies of player A and B by using Linear Programming :

$$\begin{array}{cc} & \text{Player } B \\ \text{Player } A \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 3 & -1 & 4 \\ 6 & 7 & -2 \end{bmatrix} \end{array}$$

Unit-III

3. (*a*) Solve the following integer P.P. :

Maximize $z = 2x_1 + 3x_2$

Subject to the constraints :

$-3x_1 + 7x_2 \leq 14,$

$7x_1 - 3x_2 \leq 14,$

$x_1, \; x_2 \geq 0$

and are integers

(*b*) Use branch and bound method to solve the following L.P.P. :

Minimize $z = 4x_1 + 3x_2$

Subject to the constraints :

$5x_1 + 3x_2 \geq 30,$

$x_1 \leq 4,$

$x_2 \leq 6,$

$x_1, \; x_2 \geq 0$

and are integers

(*c*) Maximize $z = x_1 + x_2$

Subject to the constraints :

$3x_1 + 2x_2 \leq 5,$

$x_2 \leq 2,$

$x_1, \; x_2 \geq 0$ and

x_1 is an integer.

Unit-IV

4. (*a*) Write the applications of operations reserach to industrial problems.

(*b*) Explain petroleum and refinery operations.

(*c*) Explain blending problems.

Unit-V

5. (*a*) Obtain the necessary and sufficient conditions for the optimum solutions of the following NLPP :

Minimize $z = f(x_1, x_2)$

$$= 3e^{2x_1+1} + 2e^{x_2+5}$$

Subject to the constraints :

$x_1 + x_2 = 7$ and

$x_1, x_2 \geq 0$

(*b*) Use Wolfe's method to solve

Max. $z = 4x_1 + 6x_2 - 2x_1^2 - 2x_1x_2 - 2x_2^2$

Subject to the constraints :

$x_1 + 2x_2 \leq 2$ and

$x_1, x_2 \geq 0$

(*c*) Solve the following quadratic programming problems by using Beale's method :

Maximize $z = 2x_1 + 3x_2 - x_1^2$

Subject to the constraints :

$x_1 + 2x_2 \leq 4$ and

$x_1, x_2 \geq 0$

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MATHEMATICS

Paper - II

Partial Differential Equations and Mechanics

Time : Three Hours] [*Maximum Marks* : 80

Note : Answer any **two** parts from each question. All questions carry equal marks.

Unit-I

1. (*a*) State and prove Hamilton ODE.

(*b*) Derive Hopf-Lax formula.

(*c*) For asymptotics in $\underline{|\infty}$ norm, there exists a constant C such that $|u(x,t)| \leq C/\sqrt{t}$.

Unit-II

2. (*a*) Use separation of variables to solve the porous medium equation

$u_t = \Delta\left(u^{\Upsilon}\right) = 0$ in $\mathbb{R}^n \times (0, \infty)$.

(*b*) State and prove Plancherel's theorem.

(*c*) Derive Hopf-Cole transformation.

Unit-III

3. (*a*) Explain about vanishing viscosity method for Burger's equation.

(*b*) Write about asymptotics for linear terms.

(*c*) Define Majorants. Show that if $f = \sum_{\alpha} f_{\alpha} \cdot x^{\alpha}$ converges for $|x| < r$ and $0 < s\sqrt{n} < r$ then f has a majorant for $|x| < s\sqrt{n}$.

Unit-IV

4. (*a*) State and prove the principle of least action.

(*b*) Explain about Poincare-Cartan integral.

(*c*) Show that the transformation

$$p = q \cot p, \quad Q = \log\left(\frac{1}{q}\sin p\right)$$

is cannonical.

Unit-V

5. (*a*) State and prove the relation between Lagrange's and Poisson's brackets.

(*b*) Prove that the Poisson bracket of two constants of motion is itself a constant of the motion.

(*c*) State and prove Jacobi Identity through Poisson bracket.

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MATHEMATICS

Optional - A

Paper - V

Programming in 'C' (with ANSI Features) - II

Time : Three Hours] [*Maximum Marks* : 70

Note : Answer any **two** parts from each question. All questions carry equal marks.

1. (*a*) What is static storage class ? Explain with suitable example.

(*b*) Demonstrate local and global variable using suitable example.

(*c*) Explain ANSI rules for the syntax and semantics of the storage class.

2. (*a*) What is Pointer ? Explain pointer arithmetics with example.

(*b*) How to pass Array as argument in a function ? Explain with suitable example.

(*c*) Demonstrate pointer to pointer with suitable example.

3. (*a*) What is recursive function ? Explain with example.

(*b*) Demonstrate macro substitution with suitable example.

(*c*) What is conditional compilation ? Explain with suitable example.

4. (*a*) Write a program to input Roll No, Name and Marks of any three subjects then calculate total marks and percentage using structure.

(*b*) Write a program to demonstrate, how the memory is allocated dynamically and release.

(*c*) Write a program to add a new node in single link list and display the value of all nodes.

5. (*a*) What is error ? How to handle errors in 'C' languages ?

(*b*) Demonstrate the reading and writting in a file with suitable example.

(*c*) Explain any five input/output functions in 'C' language with example.