Roll No. ....

DD–2803

M. A./ M. Sc. (Previous) EXAMINATION, 2020

MATHEMATICS

Paper Third

(Topology)

Time : Three Hours

Maximum Marks : 100

Note : All questions are compulsory. Solve any *two* parts of each question. All questions carry equal marks.

Unit—I

1. (a) Prove that no set can be equivalent to its power set.

   (b) If $\{T_\alpha\}_{\alpha \in \Lambda}$ is a family of topologies on a non-empty set X, then prove that (X, T) is also a topological space, where $T = \bigcap_{\alpha \in \Lambda} T_\alpha$.

   (c) Define Kuratowski closure operator on a non-empty set X. Prove that if **C** is the Kuratowski's closure operator on X, then there exists a unique topology **T** on X such that for each $A \subset X, C(A)$ coincides with T-closure of A.

Unit—II

2. (a) Let X, Y and Z be topological spaces and the mapping $f: X \to Y$ and $g: Y \to Z$ be continuous. Then prove that the composition mapping $g \circ f$: $X \to Z$ is also continuous.

   (b) Prove that a topological space (X, T) is T_1-space iff every singleton subset $\{x\}$ of X is T-closed.

   (c) State and prove Urysohn's lemma.

Unit—III

3. (a) Prove that every closed subset of a compact set is compact.

   (b) Prove that a Hausdorff space X is locally compact iff each of its points is an interior point of some compact subspace of X.

   (c) Prove that continuous image of a connected set is connected.

Unit—IV

4. (a) State and prove Tychonoff's theorem.

   (b) State and prove Embedding lemma.

   (c) Prove that the product space $X = \prod\limits_{\alpha \in \Lambda} X_\alpha$ is connected iff each coordinate space X_α is connected.

Unit—V

5. (a) Prove that the relation '$\simeq_p$' of path homotopy is an equivalence relation.

   (b) Let α be a path in X from x_0 to x_1. Define a map $\hat{\alpha}: \pi_1(X, x_0) \to \pi_1(X, x_1)$ by $\hat{\alpha}([f]) = [\bar{\alpha}] \times [f] \times [\alpha]$. Prove that $\hat{\alpha}$ is a group isomorphism.

   (c) Let (X, T) be a topological space and let $Y \subset X$. Then prove that Y is T-open iff no net in X-Y can converge to a point in Y.